

Ultracold Scattering Processes in Three-Atomic Helium Systems* [†]

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We review results on scattering observables for ^4He – $^4\text{He}_2$ and ^3He – $^4\text{He}_2$ collisions. We also study the effect of varying the coupling constant of the atom-atom interaction on the scattering length.

1. INTRODUCTION

Experimentally, ^4He dimers have been observed in 1993 by Luo *et al.* [1], and in 1994 by Schöllkopf and Toennies [2]. In the latter investigation the existence of ^4He trimers has also been demonstrated. Later on, Grisenti *et al.* [3] measured a bond length of 52 ± 4 Å for $^4\text{He}_2$, which indicates that this dimer is the largest known diatomic molecular ground state. Based on this measurement they estimated a scattering length of 104^{+8}_{-18} Å and a dimer energy of $1.1^{+0.3}_{-0.2}$ mK [3]. Further investigations concerning ^4He trimers and tetramers have been reported in [4], but with no results on size and binding energies. To the best of our knowledge, we are not aware of any experimental results on binding energy and size of the asymmetric helium trimer consisting of two ^4He and one ^3He atoms.

Many theoretical calculations of the helium three-atomic systems were performed in the past for various interatomic potentials. Variational, hyperspherical, and Faddeev-type techniques have been employed in this context [5]–[17]. It was found that the ^4He trimer has two bound states of total angular momentum zero: a ground state of about 126 mK and an excited state of about 2.28 mK. Experimentally this Efimov-type excited state has not yet been observed. It should be mentioned, however, that the year 2006 is noticeable due the first convincing experimental evidence for the Efimov effect in an ultracold gas of caesium atoms [18].

Due to the smaller mass of the ^3He atom, the ^3He – ^4He system is unbound. Nevertheless, the ^3He – $^4\text{He}_2$ trimer exists, though with a binding energy of about 14 mK. In contrast to the symmetric case, there is no excited state in the asymmetric ^3He – $^4\text{He}_2$ system.

Phase shifts of ^4He – $^4\text{He}_2$ elastic scattering at ultra-low energies have been calculated for the first time in [10, 11] both below and above the three-body threshold. An alternative *ab initio* calculation was performed in [13], but only below the threshold. The only available results on the ^3He – $^4\text{He}_2$ phase shifts were obtained in [16]. For completeness we notice that zero-range models are able to reproduce the ^4He – $^4\text{He}_2$ scattering situation [19].

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In what follows we present our results on binding energies and scattering observables in the helium three-atomic systems obtained mainly with the LM2M2 potential by Aziz and Slaman [20].

2. RESULTS

In our calculations we employed the hard-core version of the Faddeev differential equations developed in [21]. Table 1 summarizes trimer binding energies and He–He₂ scattering lengths, calculated with the LM2M2 potential. The binding energies of the ⁴He trimer ground state (E_{He_3}) and excited state ($E_{\text{He}_3}^*$) are presented in the first two rows. These results demonstrate the good agreement between the different methods.

Table 1

Results for binding energies of the ⁴He₃ and ³He⁴He₂ trimers and the He–He₂ scattering lengths.

	[11] ^a	[6]	[7]	[8]	[9]	[12]	[15]
$-E_{\text{He}_3}$ (mK)	125.9	125.2	125.5	126.41	126.39	126.4	126.39
$-E_{\text{He}_3}^*$ (mK)	2.276 ^b	2.269	2.19	2.271		2.265	2.268
$\ell_{\text{sc}}^{(^4\text{He}-^4\text{He}_2)}$ (Å)	115.5 ^c		126.	115.4 ^d			115.2
$-E_{^3\text{He}^4\text{He}_2}$ (mK)	13.84 ^e	13.66		14.4 ^f	14.165		
$\ell_{\text{sc}}^{(^3\text{He}-^4\text{He}_2)}$ (Å)	21.0 ^e			19.3 ^f			

^aCalculations with maximal value $l_{\text{max}} = 4$ of the subsystem angular momentum. ^bThis value was rounded in [11].

^cResult of extrapolation, see [14]. ^dResult from Ref. [13]. ^eResult from Ref. [16]. ^fResult from Ref. [17].

The third row shows values of the ⁴He–⁴He₂ scattering length. Notice that in Ref. [14] our previous calculations of [11] have been improved essentially by enlarging the grid parameters and the cut-off hyperradius. This table also contains the results by Blume and Greene [7], by Roudnev [13], and the most recent ones by Lazauskas and Carbonell [15]. The treatment of [7] is based on a combination of the Monte Carlo method and the hyperspherical adiabatic approach. Roudnev employs three-dimensional Faddeev differential equations. Lazauskas and Carbonell also use differential Faddeev equations, but with the hard-core boundary conditions of Ref. [21]. Our results agree rather well with these alternative calculations. For completeness we mention the model calculations of [19]. Being characterized by remarkable simplicity, they rely essentially on the binding energy obtained in *ab initio* calculations.

The last two rows of Table 1 contain results for the binding energy of the asymmetric ³He⁴He₂ trimer and for the ³He–⁴He₂ scattering length. The phase shifts $\delta(E)$ of ³He–⁴He₂ and ⁴He–⁴He₂ scattering are depicted in Fig. 1 where in both cases the normalization $\delta(0) = 0$ is used.

It is already widely accepted that the excited ⁴He trimer state is of Efimov nature. For the first time this was clearly shown by Cornelius and Glöckle [5]. A more detailed investigation has been made in [23] where the mechanism of emerging new Efimov states from resonances was studied. For a latest discussion of this subject see Ref. [15]. Table 2, which is borrowed partly from [23], demonstrates how a new Efimov level arises from a virtual state when the interatomic

Table 2

Dependence of the ^4He dimer and trimer energies (mK) and the ^4He – ^4He and ^4He – $^4\text{He}_2$ scattering lengths (\AA) on the potential strength λ . The three-body results were obtained with a maximal value $l_{\text{max}} = 0$ of the subsystem angular momentum. The HFD-B potential from Ref. [22] was used.

λ	ε_d	$\varepsilon_d - E^*$	$\varepsilon_d - E_{\text{virt}}$	$\varepsilon_d - E^{**}$	$\ell_{\text{sc}}^{(1+2)}$	$\ell_{\text{sc}}^{(1+1)}$
1.30	−199.45	-	1.831	-	−61	11.4
1.20	−99.068	-	0.01552	-	−340	14.7
1.18	−82.927	-	0.00058	-	−1783	15.8
1.17	−75.367	0.0063	-	-	8502	16.3
1.15	−61.280	0.0737	-	-	256	17.7
1.10	−32.222	0.4499	-	-	152	23.1
1.0	−1.685	0.773	-	-	160	88.6
0.995	−1.160	0.710	-	-	151	106
0.990	−0.732	0.622	-	-	143	132
0.9875	−0.555	0.222	-	-	125	151
0.985	−0.402	0.518	0.097	-	69	177
0.982	−0.251	0.447	0.022	-	−75	223
0.980	−0.170	0.396	0.009	-	−337	271
0.9775	−0.091	0.328	0.003	-	−6972	370
0.975	−0.036	0.259	-	0.002	7120	583
0.973	−0.010	0.204	-	0.006	4260	1092

interaction in the $^4\text{He}_3$ is being weakened. To this end we multiply the original HFD-B potential of Ref. [22] by a factor $\lambda < 1$. Decreasing this coupling constant, there emerges, for $\lambda \approx 0.986$, a virtual state with energy E_{virt} lying on the unphysical sheet. This energy, relative to the diatom energy ε_d , is given in the lower half of column 4. When decreasing λ further, this state turns at the value $\lambda \approx 0.976$ into the second excited state. Its energy E^{**} relative to ε_d is shown at the bottom of the fifth column. When the second excited state emerges, the atom–diatom scattering length $\ell_{\text{sc}}^{(1+2)}$ changes its sign going through a pole, while the atom-atom scattering length $\ell_{\text{sc}}^{(1+1)}$ increases monotonically.

We have also studied the opposite case where $\lambda > 1$ is increasing. In this case the scattering length $\ell_{\text{sc}}^{(1+2)}$ is growing until $\lambda \approx 1.175$. There $\ell_{\text{sc}}^{(1+2)}$ becomes negative passing through a singularity and the excited state energy E^* turns into a virtual level. The energy E_{virt} of this state is shown for $\lambda \geq 1.18$ in the upper quarter of column 4.

In Fig. 2 we graphically display the behavior of the atom-atom and atom-diatom scattering lengths $\ell_{\text{sc}}^{(1+1)}$ and $\ell_{\text{sc}}^{(1+2)}$ shown in Table 2.

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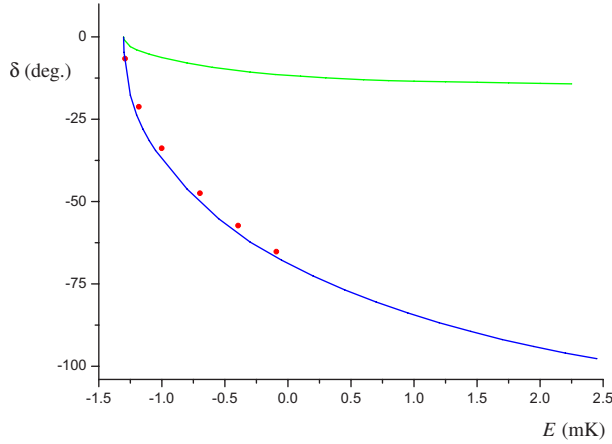


Figure 1. S-wave He–He₂ phase shifts $\delta(E)$ obtained with the LM2M2 potential. The lower curve depicts the ^4He – $^4\text{He}_2$ results of [11], the dots are the ones found in [13]. The upper curve represents the ^3He – $^4\text{He}_2$ phase shifts of Ref. [16].

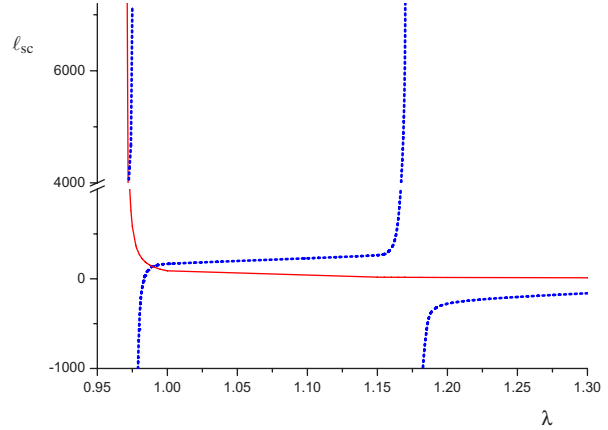


Figure 2. Dependence of the atom-atom scattering length $\ell_{\text{sc}}^{(1+1)}$ (Å) (solid curve) and atom-diatom scattering length $\ell_{\text{sc}}^{(1+2)}$ (Å) (dashed curve) on the factor λ .

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